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ABSTRACT

The amplitude of a cw signal received over Echo II varies in a random manner. This variation is caused by the surface roughness of Echo II. This report is concerned with the probability density function associated with this random phenomenon. Typical PDF curves are presented and conclusion based on the results are discussed.

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PROBABILITY DENSITY FUNCTION OF THE ENVELOPE OF A CW SIGNAL OVER ECHO II

INTRODUCTION

The possibility of using passive Echo balloons for communication purposes is currently a subject of considerable interest. The effectiveness of the Echo system as a communication link will ultimately depend on how the system affects the information sent through it. Since it has been observed that Echo II affects the signal in a random fashion [2,8,10], it will be necessary to obtain statistical data representative of Echo II before any answers are possible.

Toward this end, a cw signal (2270 mc) was sent from Collins Space Communication Facilities in Dallas, Texas and received via Echo II at The Ohio State University Satellite Communication Center. The envelope of the signal received had considerable amplitude fading. A typical sample of the received signal is shown in Fig. 1.

An important statistical parameter of the signal is the amplitude probability density function (PDF). This statistic will be the subject of

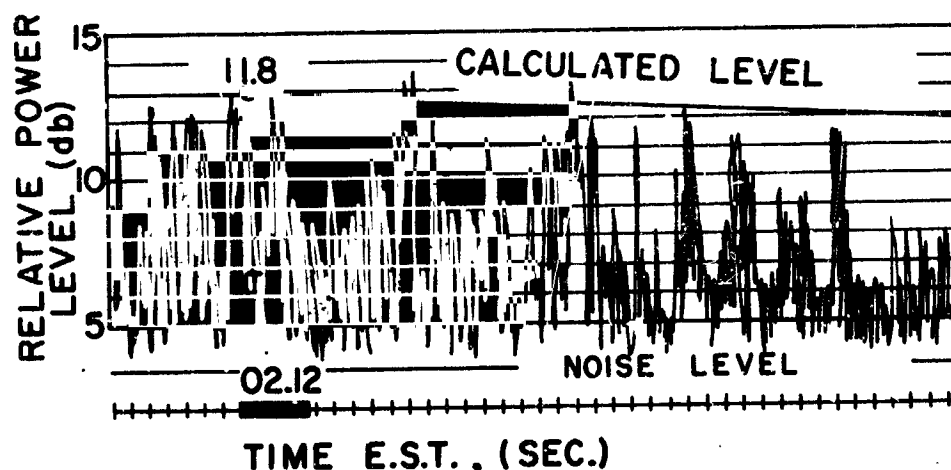


Fig. 1. Sample of the envelope of a cw signal received over Echo II communication link.

the present report. The PDF will give information on amplitude fading and surface characteristics of the Echo balloon[2,12].

The method used to measure the PDF and associated statistical errors will be discussed; other pertinent statistical considerations such as ensembles will also be considered briefly.

Typical PDF curves will be presented and some conclusions based on the results will be stated.

PROBABILITY DENSITY FUNCTION AND STATISTICAL CONSIDERATIONS

The probability density function for a continuous random variable is defined by

$$(1) \quad f(x) = \frac{d}{dx} F_X(x),$$

where $f(x)$ is the PDF and $F_X(x)$ is the probability that the random variable X is less than or equal to some number x (the distribution function[3]). The PDF can be interpreted on a time basis as follows: with reference to Fig. 2, an approximate probability density function can be defined as

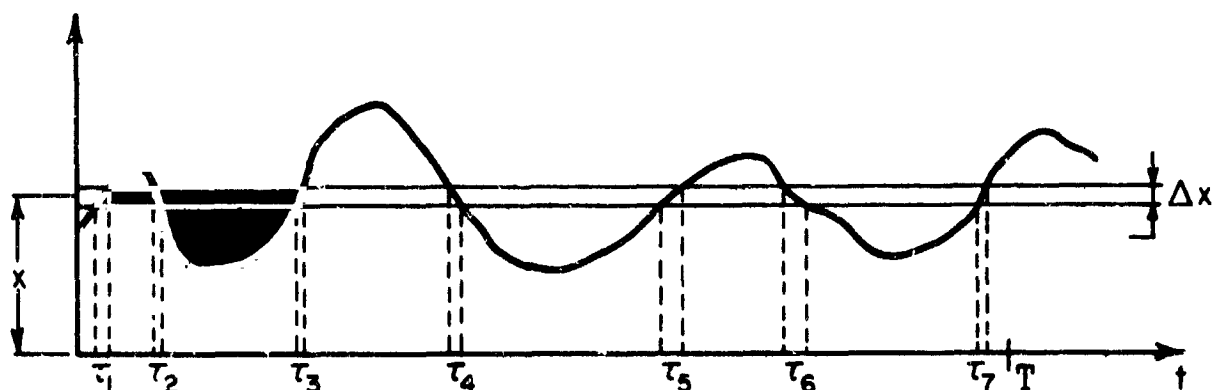


Fig. 2. Probability density function from random time function with finite sample length.

$$(2) \quad f_T(x) = \left(\frac{\sum \tau_i}{\Delta x T} \right),$$

where Δx is the window width and $\sum \tau_i$ is the total time the random variable is in the interval $x - \Delta x/2 \leq x \leq x + \Delta x/2$ over the sample length T . Taking the limit as T approaches infinity and Δx approaches zero, one has

$$(3) \quad f(x) = \lim_{T \rightarrow \infty} \left[\lim_{\Delta x \rightarrow 0} \frac{\sum \tau_i}{\Delta x T} \right].$$

In systems where sampling occurs, a probability function is defined that depends on the sampling rate ($1/\tau_s$). This function is

$$(4) \quad f_{T\tau_s}(x) = \frac{N_x}{N \Delta x},$$

where N_x is the number of sample points in the interval $x - \Delta x/2 \leq x \leq x + \Delta x/2$, and N is the total number of sample points considered ($N = T/\tau_s$). It is apparent that

$$(5) \quad f_T(x) = \lim_{\tau_s \rightarrow 0} \frac{N_x}{N \Delta x} = \lim_{\tau_s \rightarrow 0} \left(\frac{\tau_s N_x}{T \Delta x} \right) = \frac{1}{T \Delta x} \lim_{\tau_s \rightarrow 0} (\tau_s N_x).$$

Two questions naturally arise in the experimental determination of the PDF from a time basis. These are: "How fast should the sampling rate be?"; and "How long a period of time, T , is necessary for reasonable convergence?" Detailed analysis of the expected statistical errors in measurements over a finite time interval is complicated. This is true because to determine a priori the expected error in the measurement of a statistic, one must know a more complicated statistic [4,5,6]. For example, to determine the expected error in the measurement of the mean of a random process, one must know the autocorrelation function of the process [7].

In practice, a sampling rate of 10 times the reciprocal of the highest frequency will usually be more than adequate. The length of sample time T necessary for convergence is usually determined experimentally because of the lack of statistical data necessary for its calculation. Eberle[8] has shown that about 30 seconds of data are necessary for reasonable convergence of the PDF function for the envelope of the received signal from Echo II.

Another important statistical consideration is the relationship between the PDF calculated on a time basis and the PDF calculated from ensemble considerations. To answer this question, one must first define a random process that will be a reasonable mathematical model of the random phenomenon in question -- the Echo II communication link.*

One method of defining a random process that will describe an Echo II link is as follows:

Consider a collection of many Echo II links as illustrated in Fig. 3. Here the satellite is considered stationary in space with respect to

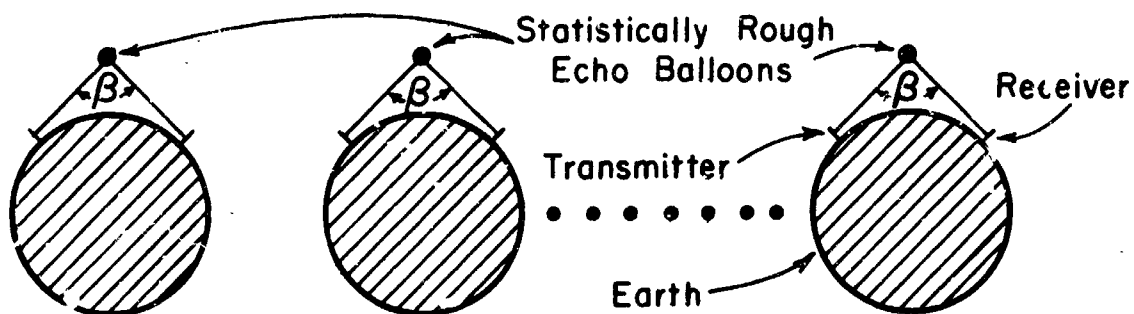


Fig. 3. A collection of Echo II communication links with fixed bistatic angle β .

* See Ref. 1 (Middleton), pages 25 to 40 for an excellent description of a random process.

transmitter and receiver. If the received signal from each link is considered as a member of an ensemble, the random process will be the collection of these signals. This is illustrated in Fig. 4. If both the

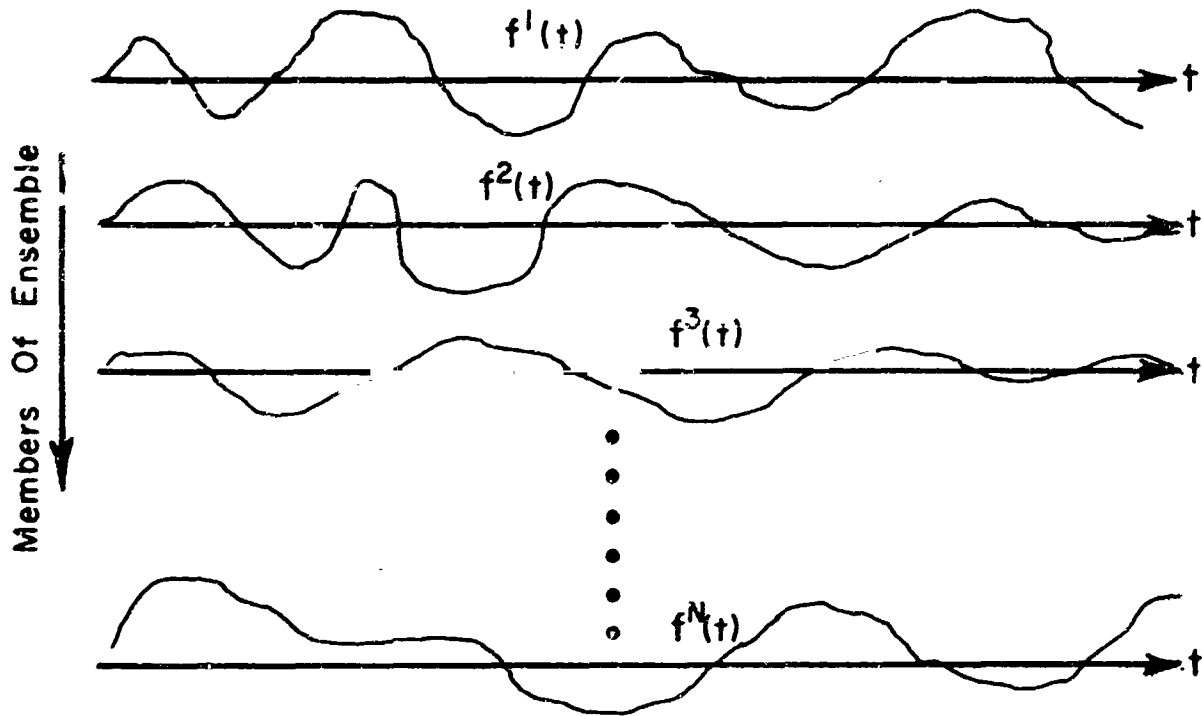


Fig. 4. Ensemble of signals received via a collection of independent Echo II communication links with fixed bistatic angle.

transmitted signal statistics and the random surface characteristics of the balloon are stationary in time, the ensemble will represent a stationary random process.

An Echo II link with a moving satellite will be described by a different random process. Consider the collection of communication links with the satellite moving from horizon to horizon (all links will have same orbit relative to transmitter and receiver so that each can be considered identical) as illustrated in Fig. 5. The random process will be the ensemble of signals received over each link. This is illustrated in Fig. 6. Even if the transmitted signal statistics and the surface roughness are stationary in time, this random process will not be stationary because the range and the bistatic angle of the scatter are changing with time. This second model is representative of Echo II as it exists today but represents only one of an infinite number of possible orbit configurations,

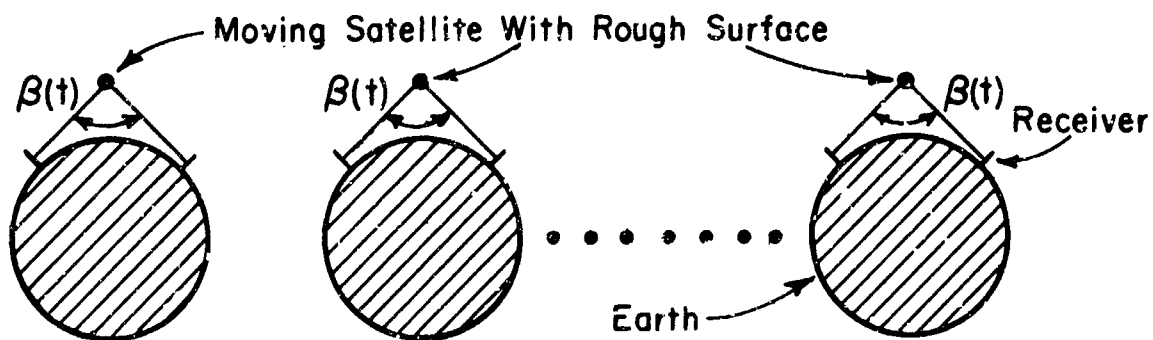


Fig. 5. A collection of Echo II links with a non-stationary bistatic angle β .

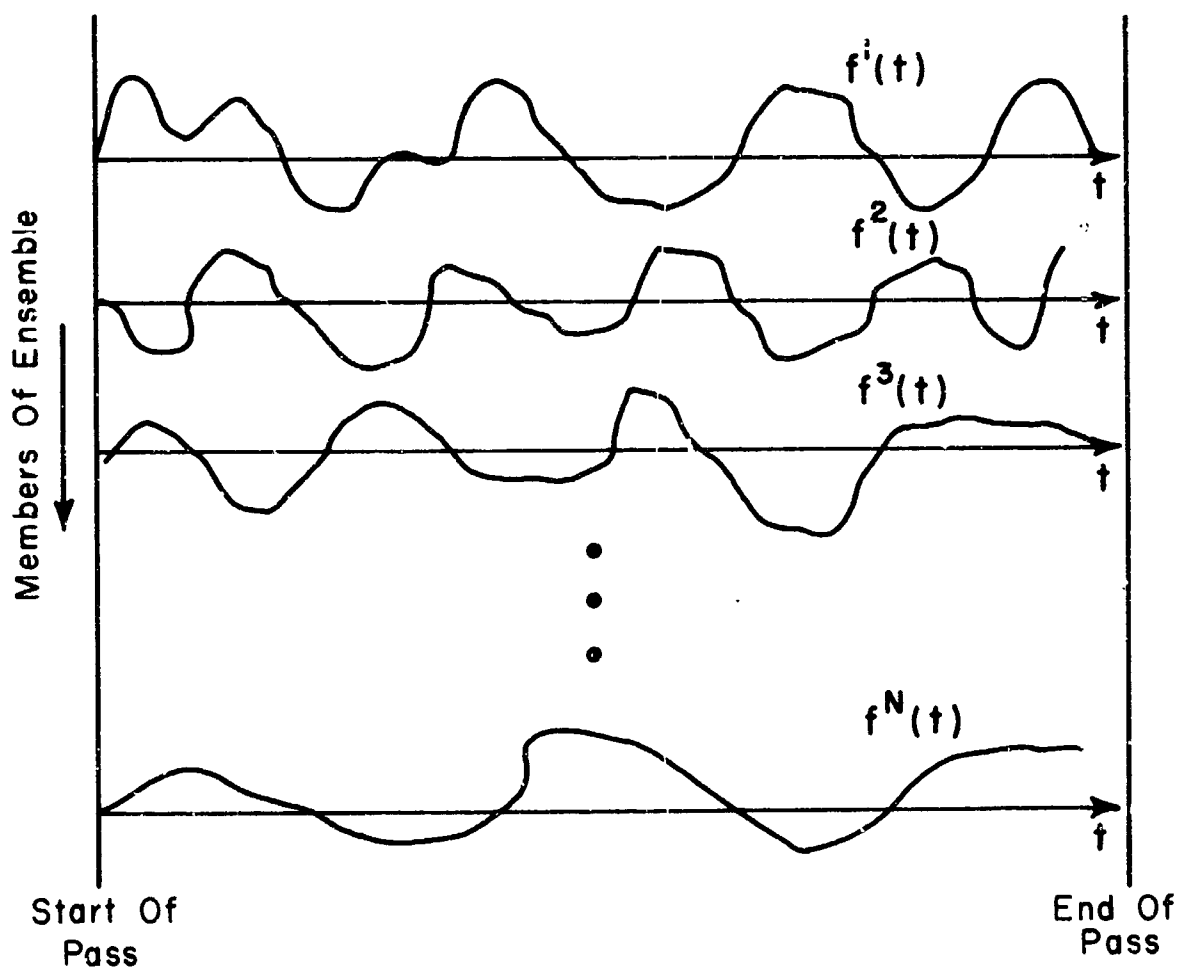


Fig. 6. Ensemble of signals received via a collection of Echo II communication links with a moving satellite.

Practically speaking, it is almost impossible to calculate the PDF from ensemble considerations. This method would require data from many Echo II passes with similar orbit configurations which would be impossible to obtain in a reasonable length of time. For this reason, the time approach was used in calculating the PDF.

Since the process is non-stationary, the lengths of the samples had to be limited. If this were not the case, the PDF from time considerations would have doubtful statistical significance from an ensemble point of view.

RESULTS

The results are presented in histogram form only. No attempt was made to draw a continuous function from the histogram because of the statistical uncertainties involved in using finite sample lengths and the lack of sufficient data necessary to arrive at any conclusions about them.

The PDFs were calculated from 30-second sample lengths of the received signal. The signals were first digitalized by hand (no machine was available to do this) and the PDF was then calculated via a computer program based on Eq. (4). (The computer program used calculates other statistics and is included in Ref. 9.)

A sampling rate of 50 times a second was selected. This rate was the highest practical value based on the condition of the received data as it was recorded. Figures 7 and 8 show the difference between the PDF as calculated by Eq. (2) (time in the window) and as calculated by Eq. (4) (the digital method).

A practical consideration that had to be taken into account was the nonlinearity of the envelope detector in the receiving equipment. Figures 8 and 9 show the difference between the PDF calculated from linear vs. nonlinear data. There is considerable difference between the two and therefore only linearized data were used. The computer program takes this into account by linearizing the digital data based on the characteristics of the particular detector used.

Figures 10, 11, 12, and 13 show the PDF for different 30-second sample lengths of Echo II pass number 2653. Figures 14, 15, 16, 17, and 18 show the PDF for different 30-second sample lengths of pass number 2816. These results are typical of the results obtained from other passes and show the strong dependence of PDF on the different sample lengths (satellite position).

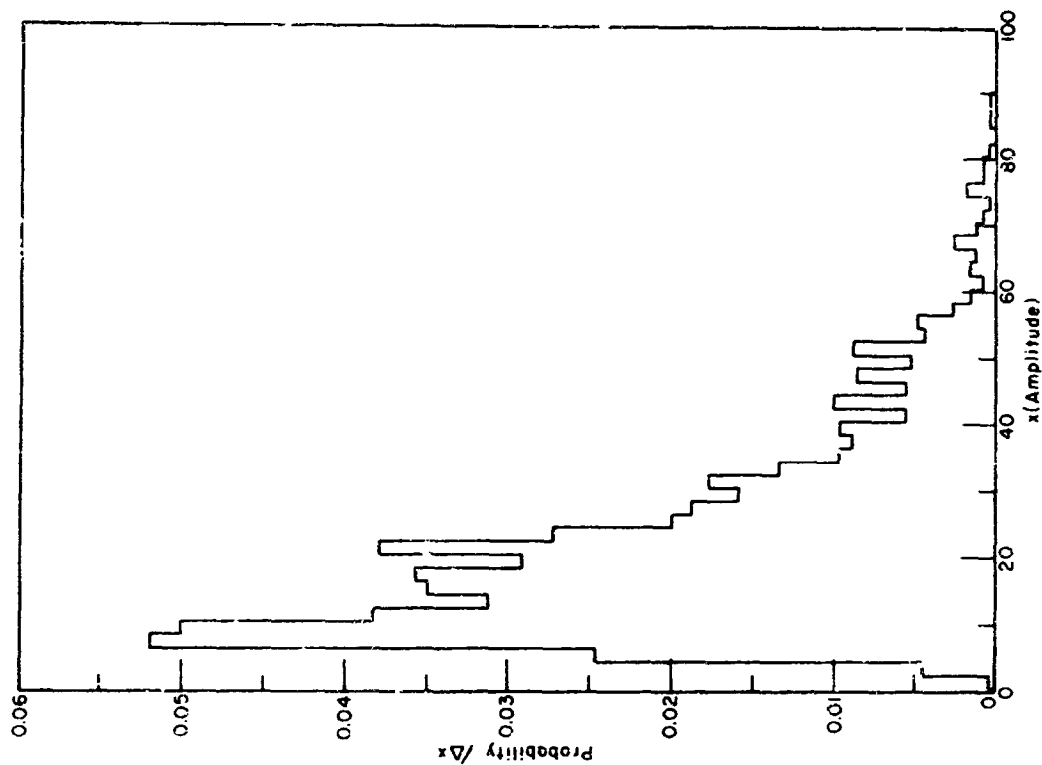


Fig. 8. Same as Fig. 7, except calculations based on digital data (50 times a second). System detector not linearized.

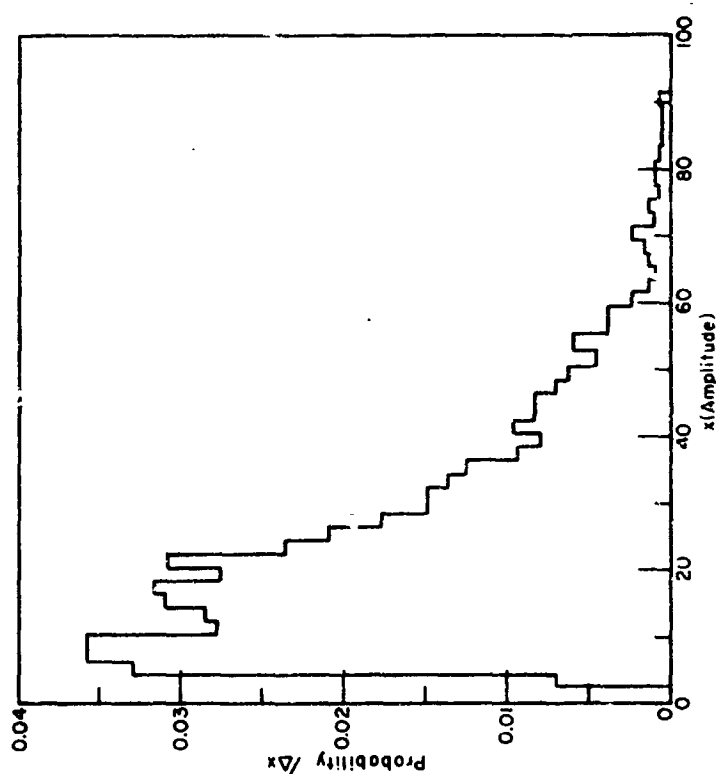


Fig. 7. Probability density function of envelope of received signal over Echo II. Cw signal from Dallas to OSU. Pass #2653. 30-second sample length. Based on time in amplitude window.

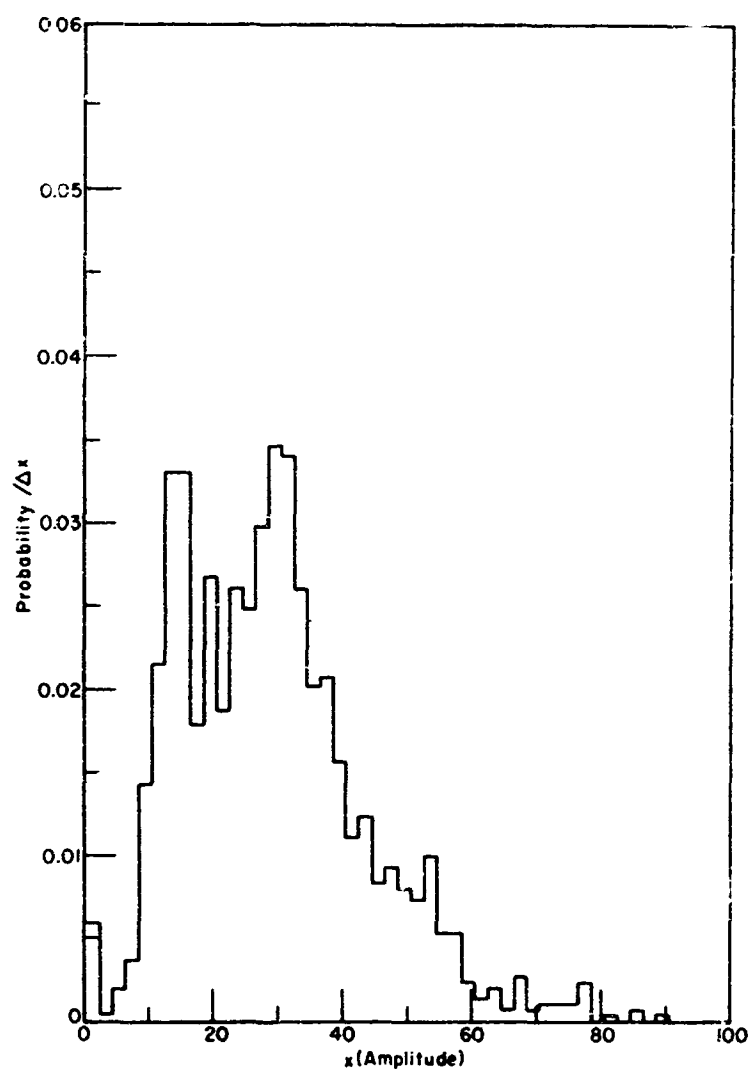


Fig. 9. Same as Fig. 7, except calculations based on digital data (50 times a second). System detector linearized.

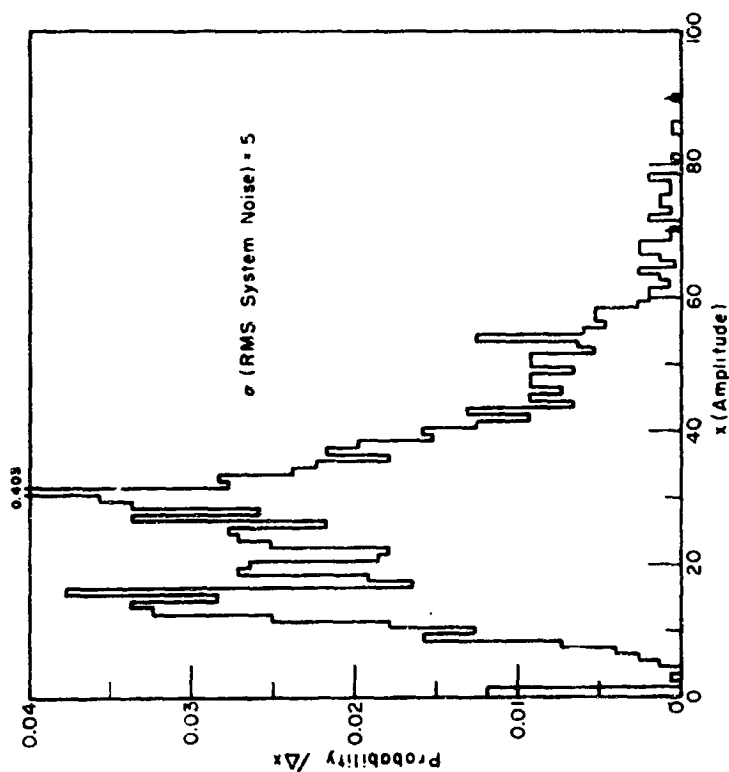


Fig. 10. Probability density function of envelope of received signal over Echo II. Cw signal from Dallas to OSU. Pass #2653. 30-second sample length. Calculations based on linearized digital data (50 times a second).

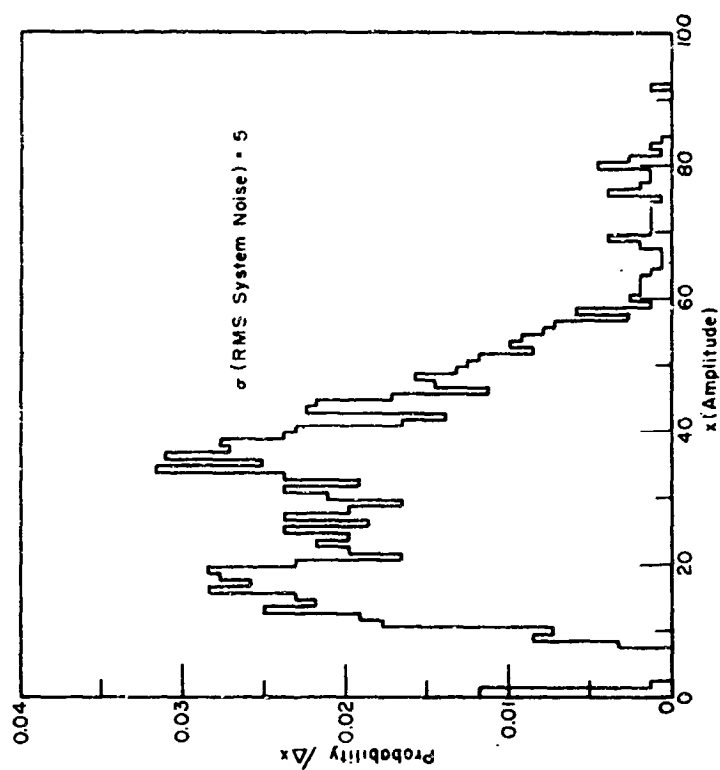


Fig. 11. Same as Fig. 10, except next 30 seconds of data used for calculations.

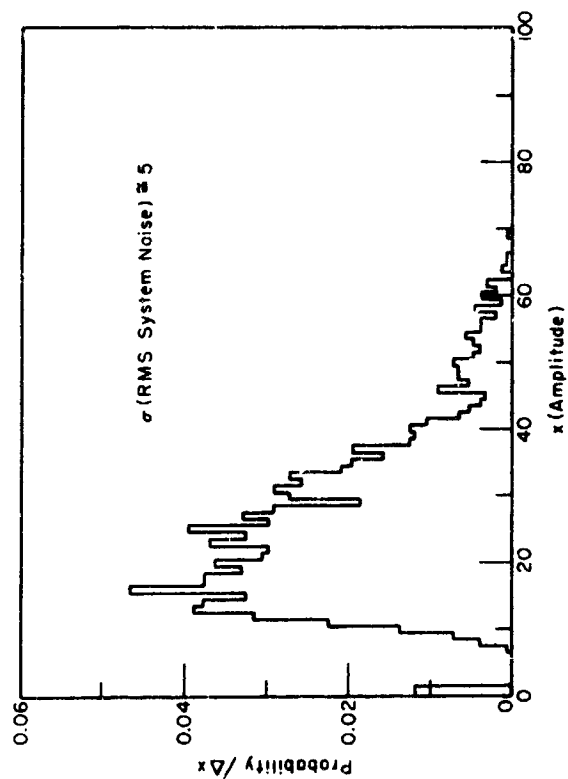


Fig. 12. Same as Fig. 10, except next 30 seconds of data used for calculations.

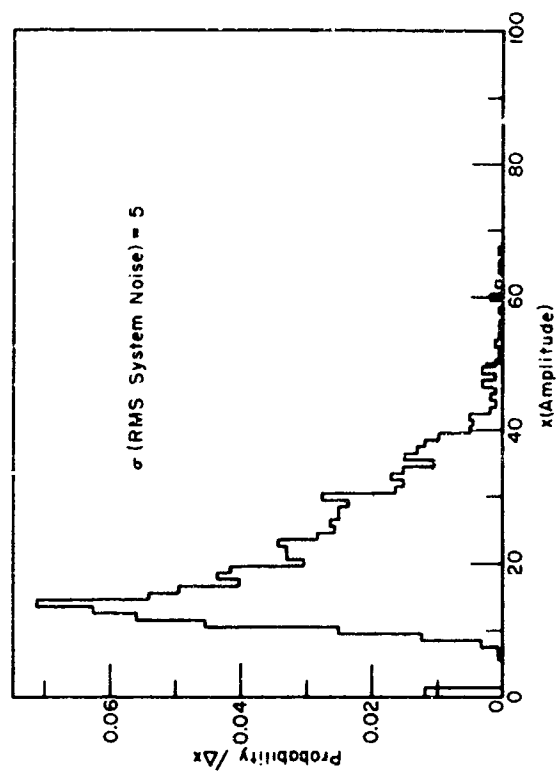


Fig. 13. Same as Fig. 10, except next 30 seconds of data used for calculations.

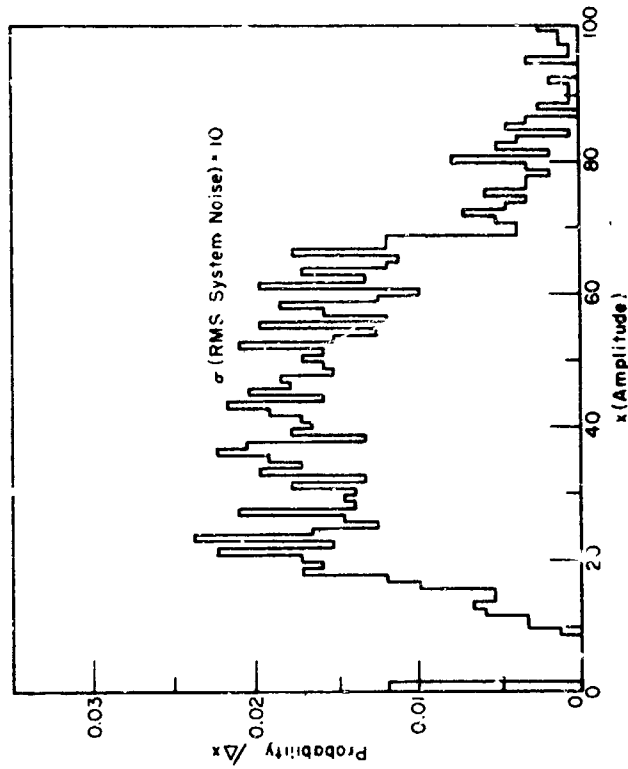


Fig. 14. Probability density function of envelope of received signal over Echo II. Cw signal from Dallas to OSU. Pass #2816. 30-second sample length. Calculations based on linearized digital data (50 times a second).

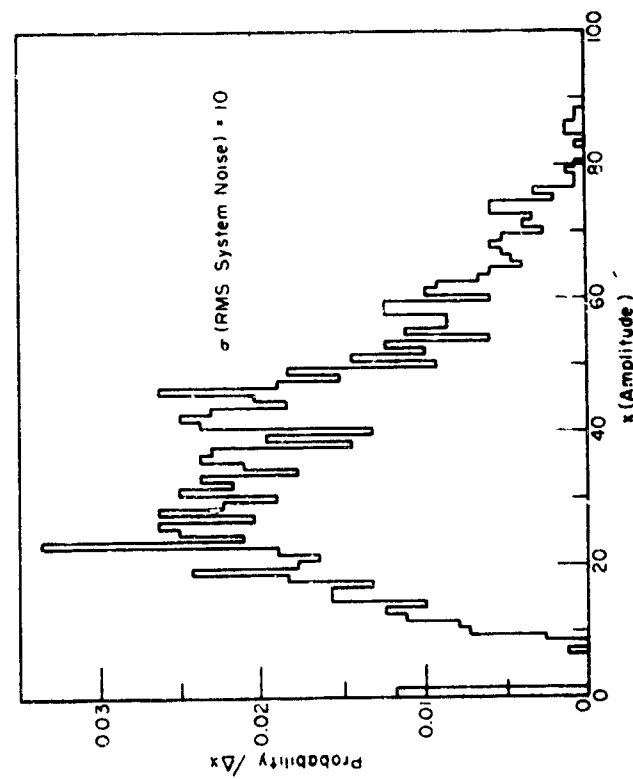


Fig. 15. Same as Fig. 14, except next 30 seconds of data used for calculations.

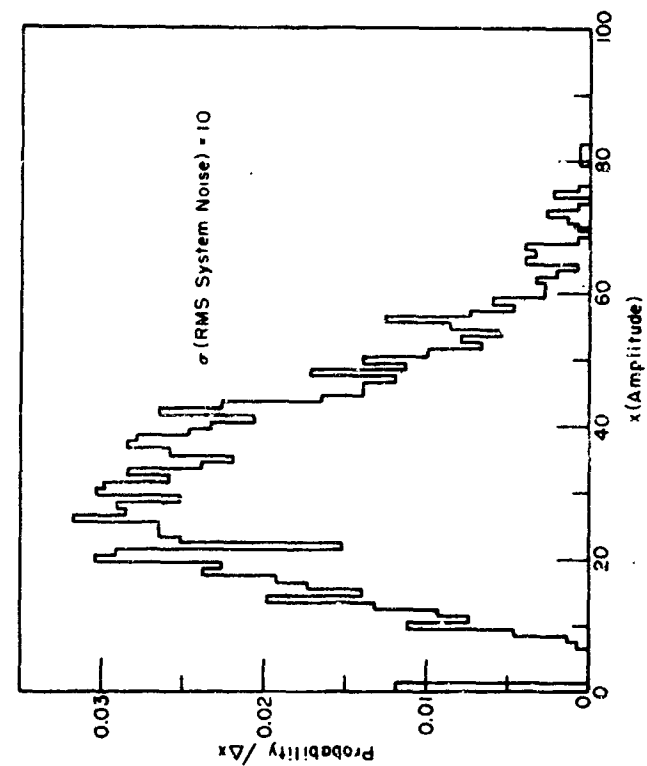


Fig. 16. Same as Fig. 14, except next 30 seconds of data used for calculations.

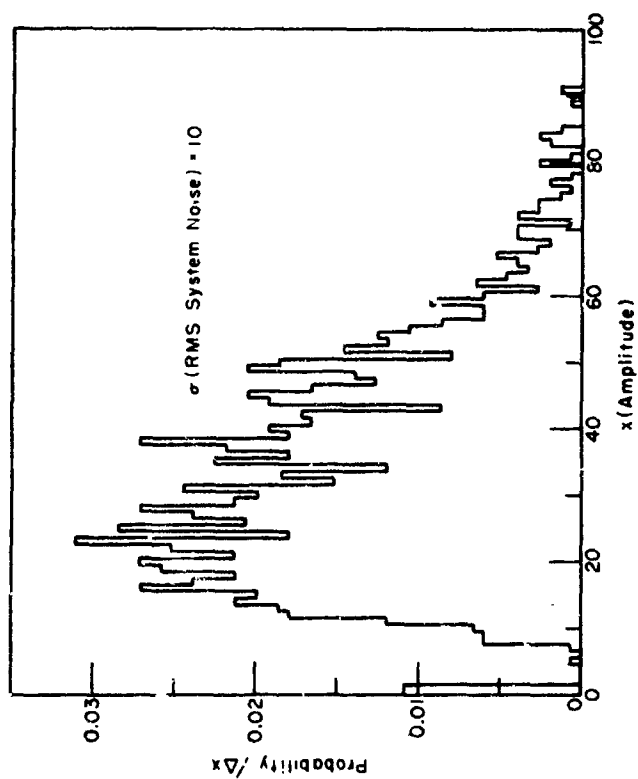


Fig. 17. Same as Fig. 14, except next 30 seconds of data used for calculations.

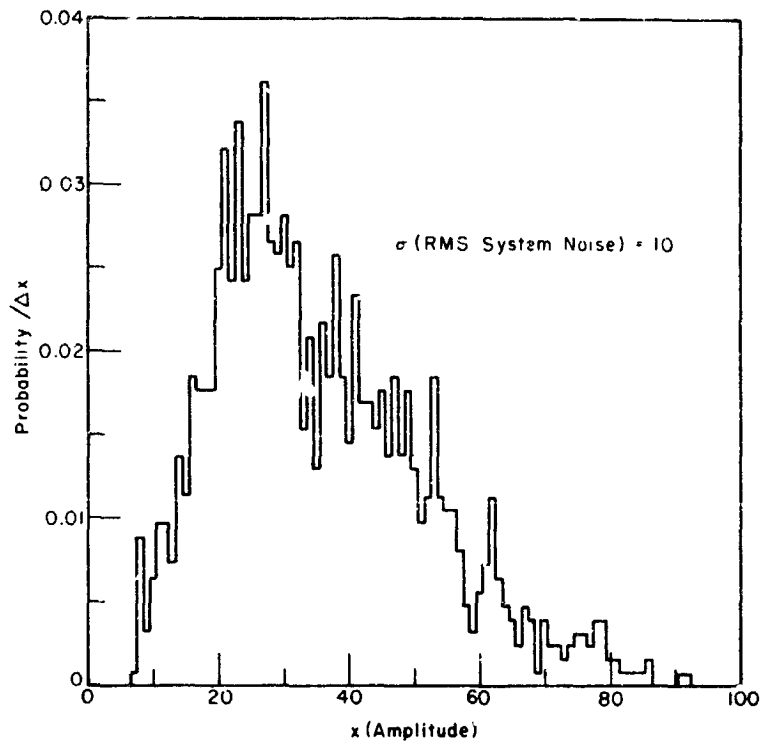


Fig. 18. Same as Fig. 14, except next 30 seconds of data used for calculations.

CONCLUSIONS

Although only a limited amount of data has been considered, a few significant conclusions can be reached concerning the envelope of the received signal over Echo II.

Comparing Figs. 7 and 8, one can conclude that a sampling rate of 50 times a second is not quite adequate to reproduce the envelope. This would indicate significant frequency components in the 10-20 cps range, and is in agreement with the power spectral density curves for the envelope of a cw signal from Echo II pass numbers 1197 and 1558. These curves are presented in Ref. 10.

The difference between the various PDFs substantiate the non-stationary property of the process of receiving a signal over Echo II. This property is due to the movement of the satellite.

The shapes of all of the PDF curves appear to be similar; to a fair approximation each PDF has a Rayleigh shape. If the return signal is thought of as a combination of a specular component (return from the

flare spot of the balloon) and many scattered components (return from the surface irregularities)[2], one can represent the signal as

$$(6) \quad A \cos \omega_{ct} + \sum_{i=1}^N A_i \cos (\omega_{ct} + \theta_i) .$$

A represents the amplitude of the specular return and A_i and θ_i are the random amplitude and phase of the i^{th} component of the scattered return. If the scattered return is thought of as an equivalent noise voltage with a Gaussian distribution, the Rayleigh shape PDF for the envelope will result[11]. Thus one can infer that the scattered component is equivalent to Gaussian noise and is of significant level compared to the specular return. (If the level was small compared to the specular return, then the PDF shape would be Gaussian[11]).

In Ref. 2, the PDF for a cw pass from Ohio University in Athens, Ohio, to Ohio State University (Echo II pass number 1901) has more of a Gaussian shape than the PDFs shown here. This would be the case if pass 1901 had a higher specular-to-scattered ratio than the passes analyzed here. Based on this comparison, one would conclude that Echo II deteriorated to some extent during the time between pass 1901 and 2653. Since many variables are unknown (for example, the difference between the two transmitters' spectrums), this conclusion may be questionable.

It is realized that many statistical questions which were raised earlier have been left unanswered. Unfortunately the data available are not sufficient to help with these considerations. The effect of using a finite sample length to calculate the PDF is presently being studied (along with several other statistical questions) and will be discussed in a future report.

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